No new expressions are obtained by subtracting these from unito or by inverting them. The study of equality among these functions becomes a special case when the group generated by the operations $x_1 - n$, x_1^2/n transforms a point into less than six distinct points. This question has been completely solved for the general dihedral rotation group.

THE EXPRESSION OF THE AREAS OF POLYGONS IN DETERMINANT FORM.

By R. P. BAKER.

The area of the triangle the rectangular coördinates of whose vertices are (x_1, y_1); (x_2, y_2); (x_3, y_3), is

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

For the area of a quadrilateral whose vertices are 1, 2, 3, 4, diagonals (13) and (24) and such that circuits (123), (134) have the area on the left, we have

$$\frac{1}{2}\begin{vmatrix} x_1, & y_1, & 1, & 0 \\ x_2, & y_2, & 1, & 1 \\ x_3, & y_3, & 1, & 0 \\ x_4, & y_4, & 1, & 1 \end{vmatrix} \equiv \frac{1}{2}\begin{vmatrix} x_1, & y_1, & 1, & -k \\ x_2, & y_2, & 1, & 1-k \\ x_3, & y_3, & 1, & -k \\ x_4, & y_4, & 1, & 1-k \end{vmatrix}.$$

The case of the pentagon or polygon of more sides than 5 is different.

Suppose that the area P of the pentagon can be expressed by

$$\lambda\begin{vmatrix} x_1, & y_1, & 1, & a_1, & b_1 \\ x_2, & y_2, & 1, & a_2, & b_2 \\ x_3, & y_3, & 1, & a_3, & b_3 \\ x_4, & y_4, & 1, & a_3, & b_4 \\ x_5, & y_5, & 1, & a_5, & b_5 \end{vmatrix} = \lambda\begin{vmatrix} x_1, & y_1, & 1, & 0, & 0 \\ x_2, & y_2, & 1, & a'_2, & b'_2 \\ x_3, & y_3, & 1, & a'_3, & b'_3 \\ x_4, & y_4, & 1, & a'_4, & b'_4 \\ x_5, & y_5, & 1, & a'_5, & b'_5 \end{vmatrix}$$

the latter being obtained from the former by subtracting multiples of columns.

Expanding by Laplace's method in minors of the first three columns we get the area as a sum of multiples of triangular areas all having the point 1 as vertex. The multipliers must obviously be equal. Hence all the determinants $\begin{vmatrix} a'_i & b'_i \\ a'_j & b'_j \end{vmatrix}$ must be equal. This is impossible, for

$$\frac{1}{2}\left|\begin{array}{cccc} a_2, & b_2, & a_2, & b_2 \\ a_2, & b_3, & a_3, & b_3 \\ a_4, & b_4, & a_4, & b_4 \\ a_5, & b_5, & a_5, & b_5 \end{array}\right| \equiv (23)(45)-(24)(35)+(25)(34)\equiv 0,$$

which cannot be satisfied by

$$(23)=(45)=(24)=(35)=(25)=(34).$$

The general case fails in consequence of a similar identical relation among the determinants of a matrix. This relation can be expressed as the expansion of a determinant of $2(n-3)$ rows which can be symbolized by $\dfrac{A \mid C}{B \mid C}$, where A, B, C denote, respectively,

$$\left|\begin{array}{cccc} 0, & 0, & \cdots\cdots, & a_{n-2} \\ 0, & 0, & \cdots\cdots, & b_{n-2} \\ \cdots & \cdots & & \cdots \\ 0, & 0, & \cdots\cdots, & l_{n-2} \end{array}\right|, \quad \left|\begin{array}{ccccc} a_2, & a_3, & a_4, & \cdots\cdots, & a_{n-2} \\ b_2, & b_3, & b_4, & \cdots\cdots, & b_{n-2} \\ \cdots & \cdots & \cdots & & \cdots \\ l_2, & l_3, & l_4, & \cdots\cdots, & l_{n-2} \end{array}\right|, \quad \left|\begin{array}{cccccc} a_{n-1}, & a_n, & a_2, & a_3, & \cdots\cdots, & a_{n-4} \\ b_{n-1}, & b_n, & b_2, & b_3, & \cdots\cdots, & b_{n-4} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ l_{n-1}, & l_n, & l_2, & l_3, & \cdots\cdots, & l_{n-4} \end{array}\right|.$$

When this is expanded according to Laplace's method in determinants of $(n-3)$ rows, we get

$$\begin{aligned} &(2,\ 3,\ \cdots\cdots,\ n-3,\ n-2)(n-1,\ n,\ 2,\ \cdots\cdots,\ n-4) \\ -&(2,\ 3,\ \cdots\cdots,\ n-3,\ n-1)(n-2,\ n,\ 2,\ \cdots\cdots,\ n-4) \\ +&(2,\ 3,\ \cdots\cdots\cdots\cdots,\ n-3,\ n)(n-3,\ n,\ 2,\ \cdots\cdots,\ n-4)=0, \end{aligned}$$

which cannot be satisfied if these determinants are all equal.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Problem 206 was also solved by J. E. Sanders; No. 207 was also solved by L. E. Newcomb.

208. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Solve (1) .... $x^4+y^4=14x^2y^2$; (2) .... $x+y=m$.

I. Solution by EDWIN L. RICH, Student at Lehigh University.

Equation (1) may be written

$$(x^2-y^2-xy\sqrt{12})(x^2-y^2+xy\sqrt{12})=0 \ \ldots\ldots\ldots (3).$$